

Решения на задачи от изпита по Модул 1, проведен на 1 Май 2010 г.

Зад.1

Отг. Б

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Зад.2

Акумулираната сума =

$$= 1000 \times (1 + 2i_1) \times (1 + i_2)^2 = 1000 \times 1.16 \times \left(\frac{1}{1 - d_2^{(12)}/12} \right)^{24} = \frac{1000 \times 1.16}{0.995^{24}} \approx 1308.29$$

Отг. Б

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Зад.3

$$(i) \quad AV = 1000 \times \exp\left(\int_0^4 \delta(t) dt\right) = 1000 \times \exp\left(\int_0^3 0.02t dt + \int_3^4 0.045 dt\right) = \\ = 1000 \times \exp(0.01t^2 \Big|_0^3 + 0.045t \Big|_3^4) = 1000 \times e^{0.135} \approx 1144.54. \text{ Отг. В}$$

$$(ii) \quad 1000 \times \left(1 + \frac{j^{(4)}}{4}\right)^{16} = 1144.54$$

$$j^{(4)} = 4 \times \left(\sqrt[16]{\frac{1144.54}{1000}} - 1\right) \approx 0.034. \text{ Отг. А}$$

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Зад.4

Натрупаната сума във фонда на Иван = $10 \times 1.085^{10} + 30 \times 1.085^5 = 67.72$

Натрупана сума във фонда на Мария = $10 \times 1.0915^{10-n} + 30 \times 1.0915^{10-2n} =$

$$= 10 \times 1.0915^{10} \times v^n + 30 \times 1.0915^{10} \times v^{2n} \approx 24 \times v^n + 72 \times v^{2n}$$

$$24 \times v^n + 72 \times v^{2n} = 67.72$$

$$v^n = \frac{-24 \pm \sqrt{576 + 19503}}{144} \approx 0.8174$$

$$n \times \ln(1/1.0915) = \ln(0.8174)$$

$n \approx 2.3$. Отг. Б

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Зад.5

Настоящата стойност на първия перпетуитет е равна на:

$$\ddot{a}_{\infty|}^{(2)} = \frac{2}{d^{(2)}} = 20 \Rightarrow d^{(2)} = 0.10$$

$$d = 1 - \left(1 - \frac{0.10}{2}\right)^2 \approx 0.0975$$

$$v = 1 - d = 0.09025$$

Настоящата стойност на втория перпетуитет е равна на:

$$X \times (1 + v^2 + v^4 + v^6 + \dots) = 20$$

$$\frac{X}{1 - v^2} = 20 \Rightarrow X = 20 \times (1 - v^2) \approx 3.71. \text{ Отг. В}$$

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Зад.6

Настоящата стойност на плащанията е:

$$\ddot{a}_{\overline{n}|} = 1 + v + v^2 + \dots + v^{n-1} = \frac{1 - v^n}{d} = \frac{1 - v^n}{1 - e^{-\delta}}. \text{ Отг. А}$$

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Зад.7

$$(i) \frac{10000}{1 + 8/12 \times 2\%} \approx 9868.42. \text{ Отг. В}$$

$$(ii) 10000 \times (1 - 8/12 \times 3\%) = 9800.00. \text{ Отг. А}$$

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Зад.8

$$(i) {}_2q_{[50]} = \frac{d_{[50]} + d_{[50]+1}}{l_{[50]}} \Rightarrow l_{[50]} = \frac{11 + 16}{0.003} = 9000$$

$$\text{От } d_{[50]} + d_{[50]+1} = l_{[50]} - l_{52}, \text{ получаваме } l_{52} = 9000 - (11 + 16) = 8973. \text{ Отг. А}$$

$$(ii) p_{50} = 1 - q_{50} = 1 - q_{[50]} / 60\% = 1 - \frac{d_{[50]}}{0.6 \times l_{[50]}} = 1 - \frac{11}{0.60 \times 9000} = 0.99796. \text{ Отг. Б}$$

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Зад.9

$${}_{0.25}q_{50.25} = \frac{l_{50.25} - l_{50.50}}{l_{50.25}}$$

$$l_{50.25} = (1 - 0.25) \times l_{50} + 0.25 \times l_{51} = 0.75 \times 32669.855 + 0.25 \times 32513.405 \approx 32630.74$$

$$l_{50.50} = (1 - 0.5) \times l_{50} + 0.5 \times l_{51} = 0.5 \times 32669.855 + 0.5 \times 32513.405 \approx 32591.63$$

$${}_{0.25}q_{50.25} = \frac{32630.74 - 32591.63}{32630.74} \approx 0.0011986. \text{ Отг. В}$$

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Зад.10

$$\frac{l_x}{l_{50}} = \frac{10}{100} \Rightarrow l_x = 0.1 \times l_{50} = 9008.5$$

Тъй като $l_{85} = 10169$ и $l_{86} = 8271.6$, следователно x е между 85 и 86 години. Отг. Б

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Зад.11

Отг. В

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Зад.12

$${}_0e_{40} = \int_0^{60} {}_t p_{40} dt = \int_0^{60} \left(1 - \frac{t}{60}\right)^{0.5} dt = \left[-60 \times \left(1 - \frac{t}{60}\right)^{1.5} \times \frac{2}{3} \right]_0^{60} = 60 \times \frac{2}{3} = 40. \text{ Отг. Г}$$

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Зад.13

$${}_2q_{40} = 1 - \frac{l_{42}}{l_{40}} = 1 - \frac{33439.528}{33542.311} = 0.00306$$

$${}_{|1}q_{40} = \frac{l_{41} - l_{42}}{l_{40}} = \frac{33493.920 - 33439.528}{33542.311} = 0.00162$$

$${}_{60}p_{40} = \frac{l_{100}}{l_{40}} = \frac{72.353686}{33542.311} = 0.00216$$

$${}_{58|2}q_{40} = \frac{l_{98} - l_{100}}{l_{40}} = \frac{187.72094 - 72.353686}{33542.311} = 0.00344.$$

Отг. Б

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Зад.14

Отг. А

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Зад.15

$$A_{[x:\overline{n}]}^1 = A_{[x]} - v^n \times \frac{D_{x+n}}{D_{[x]}} \times A_{x+n}$$

$$A_{[60]} = 1 - d \times \ddot{a}_{[60]} = 1 - d \times (1 + a_{[60]}) = 1 - \frac{0.08}{1.08} \times (1 + 9.326) \approx 0.2351$$

$$A_{70} = 1 - d \times \ddot{a}_{70} = 1 - d \times (1 + a_{70}) = 1 - \frac{0.08}{1.08} \times (1 + 7.308) \approx 0.3846$$

$$A_{[60]:10}^1 = 0.2351 - \frac{1}{1.08^{10}} \times \frac{780683}{893918} \times 0.3846 \approx 0.0795 . \text{ Отг. А}$$

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Зад.16

Отг. Г

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Зад.17

$$A_{x:\overline{20}|} = A_x - A_{x:\overline{20}|}^1 \times A_{x+20} + A_{x:\overline{20}|}^1$$

$$0.55 = 0.25 - A_{x:\overline{20}|}^1 \times 0.40 + A_{x:\overline{20}|}^1$$

$$0.30 = 0.60 \times A_{x:\overline{20}|}^1$$

$$A_{x:\overline{20}|}^1 = 0.50 . \text{ Отг. В}$$

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Зад.18

$$(i) \quad SP = \frac{C_{30} + C_{31}}{D_{30}} \times 10000 = \frac{6.5577344 + 6.4684530}{10433.31} \approx 12.49 . \text{ Отг. Г}$$

$$(ii) \quad SP^* = (q_{30}^* \times v + p_{30}^* \times q_{31}^* \times v^2) \times 10000 =$$

$$= \left[(1 - p_{30}^5) \times v + p_{30}^5 \times (1 - p_{31}^5) \times v^2 \right] \times 10000 =$$

$$= \left[\frac{1 - (1 - 0.00065368)^5}{1.04} + \frac{(1 - 0.00065368)^5 \times (1 - (1 - 0.00067101)^5)}{1.04^2} \right] \times 10000 =$$

$$= \left[\frac{1 - 0.996736}{1.04} + \frac{0.996736 \times (1 - 0.99665)}{1.0816} \right] \times 10000 \approx 62.26 . \text{ Отг. А}$$

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Зад.19

$$1000A_{60} = v \times 1000q_{60} + v \times p_{60} \times 1000A_{61} = \frac{1}{1.06} \times (20 + 0.980 \times 440) \approx 425.66$$

$$\ddot{a}_{60} = \frac{1 - A_{60}}{d} \approx \frac{1 - 0.42566}{0.06/1.06} \approx 10.14667$$

$$1000 {}_{10}V_{50} = 1000A_{60} - 1000P_{50} \times \ddot{a}_{60} \approx 425.66 - 25 \times 10.14667 \approx 172. \text{ Отг. Б}$$

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Зад.20

$$(i) \quad E[Y_{65}] = \ddot{a}_{65}, \quad E[Y_{75}] = \ddot{a}_{75}$$

$$Var[Y_{65}] = \frac{1}{d^2} ({}^2A_{65} - A_{65}^2) = \frac{1}{(0.05660)^2} (0.23603 - (0.4398)^2) \approx 13.2996. \text{ Отг. Б}$$

$$(ii) \quad Var[Y_{75}] = \frac{1}{d^2} ({}^2A_{75} - A_{75}^2) = \frac{1}{(0.05660)^2} (0.38681 - (0.59149)^2) \approx 11.5339. \text{ Отг. Б}$$

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